MathsCraft
Doing maths like a research mathematician

MATHSCRAFT CURRICULUM

Australian Government
Australian Research Council

ACEMS

MathsCraft is supported by the ARC Centre of Excellence for Mathematical & Statistical Frontiers (ACEMS) with partner researchers from the University of Melbourne, the University of Adelaide, Queensland University of Technology, University of Technology Sydney, the University of Queensland, UNSW Sydney, and Monash University.

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BACKGROUND

Mathematics is a set of connected *ideas* that has enabled humans to solve many problems and create many things of value to our lives.

Doing mathematics is a process where *ideas that come to mind* are sequenced in unfamiliar chains, in order to *explore, solve, create and understand* the previously unexplored, previously unsolved, yet to be created and not yet understood. *Doing mathematics* is an adventure, often quite an exciting adventure.

Ideas, in this context, refer to *facts, concepts, processes and/or associations*.

The value of mathematics to our lives is not in question. What remains in question, after decades of effort, is how to positively engage school-aged people with mathematics, so they have the best chance of *doing mathematics*, and thereby positively influencing their own lives and the lives of those around them.

Despite the introduction of many alternative ways of engaging students with mathematics at school, over many decades, the number of students studying higher levels of mathematics continues to fall and the claims of widespread disengagement continue.

Most people who *do mathematics* will tell you it is engaging, wonderfully satisfying, challenging and fulfilling, but not easy. As such, it seems sensible that if school students were engaged in *doing mathematics*, while continuing to learn new skills and processes at a necessary rate, we might be part way home to quelling the fall in numbers and the disengagement. But how?

Doing mathematics is made much harder if the doer does not readily *recall* the ideas they need to recall or, if they can recall them, are then unsure about how to *use* them. In other words, *doing mathematics* is hard for an individual if they are not *operational* with the facts, concepts, processes and associations that are required.

So, could positive engagement be as simple as ensuring students are operational with the required facts, concepts, processes and associations when *doing mathematics*? That is, engaging students with the:

- previously *unexplored* (by them),
- previously *unsolved* (by them),
- yet to be *created* (by them), and
- not yet *understood* (by them),

but being as sure as possible that the student will be able to recognise the need for an idea, and then use it with confidence.

Such a situation is not dissimilar to the way research mathematicians work. Research mathematicians use mathematics with which they are operational, to work on problems that are previously unsolved and not yet understood by anyone. The solving of a problem often leads to new ideas and new problems resulting in what we term mathematical *adventures*.

Imagine if students had the opportunity to *do mathematics* like a research mathematician, in areas that have been unexplored by them. This is exactly what MathsCraft is all about. There are an amazing number of simply stated problems that lead to delightful *adventures* that only require ideas with which students are already operational.

MathsCraft: Doing Maths like a Research Mathematician is a program that offers all students the opportunity to use *facts, concepts, processes and associations* with which they are operational, in

authentic, engaging and challenging adventures, that develop their ability to explore, solve, create and understand. In doing so, the joy of *doing mathematics* is nurtured and our experience suggests that it drives interest in, and provides a purpose for, learning new mathematical knowledge and skills.

While a problem is the doorway to a MathsCraft adventure, MathsCraft is not just a set of problems. *Doing mathematics* requires the doer to behave in certain ways and MathsCraft allows students to learn and practice these critical behaviours. Since *doing mathematics* requires facts, concepts, processes and associations with which the doer is already operational, MathsCraft is not an alternative way to learn new skills and knowledge. Therefore, not all the time devoted to school mathematics should be devoted to MathsCraft; a considerable amount needs to be devoted to ensuring students learn the facts, concepts, processes and associations they will need to *do mathematics* in the future.

One way to incorporate MathsCraft into school mathematics is through the MathsCraft Curriculum.

BEING OPERATIONAL: A KEY CONCEPT

For someone to be *operational with* a fact, concept, process or association means two things:

- it can come to mind prompted only by the problem being considered, and independently of it being recently taught, and
- the person is confident in its use and can use it to good effect.

Becoming operational with a fact, concept, process or association requires experience; practice and time. The required amount of practice and time varies from person to person. It can be over a period of years.

Our experience suggests that if, in order to solve a problem, students require ideas to come to mind that have only recently been taught, then only a small proportion of students will positively engage. However, if sufficient time has passed since the facts, concepts, processes and associations were first taught, more students will be operational with them, and a higher proportion of the students will be able to naturally engage with problems that require their recall.

A MathsCraft adventure is therefore considered to be appropriate for a particular student if they are operational with the facts, concepts, processes and associations required. For many students, this is most likely if the facts, concepts, processes and associations were first learned some time before they engage with the adventure.

This time-lag, between the learning of facts, concepts, processes and associations and needing them to do mathematics, gives more students the chance to find *doing mathematics* engaging, wonderfully satisfying, challenging and fulfilling (but not easy).

- MathsCraft is not just a set of problems.
- MathsCraft is a way to engage students with *doing mathematics*, using *facts, concepts, processes and associations* with which they are already *operational*.
- Students start with a *problem*, which results in an *adventure* during which they *explore, solve, create and understand*.
- MathsCraft models a set of *behaviours* and provides the opportunity to develop those behaviours.
- MathsCraft is not an alternative way of learning new skills and knowledge.
- Some of the time devoted to school mathematics should be allocated to MathsCraft. A considerable amount of time should be allocated to learning *facts, concepts, processes and associations*.
- One way to incorporate MathsCraft into school mathematics is through the MathsCraft Curriculum.

GENERAL OVERVIEW

The MathsCraft Curriculum is a companion curriculum to the Australian Curriculum: Mathematics, and hence all local implementations. It is a structured program that supports teachers to successfully engage students in doing mathematics.

The MathsCraft Curriculum is designed for all students in Years 5–10.

The MathsCraft Curriculum comes in two versions: Curriculum A and Curriculum B. The key difference between the two curricula is that Curriculum A consists of problems that provide a narrower scope of adventure, when compared to Curriculum B.

Curriculum A requires a minimum of 6 hours of class time.
Curriculum B requires a minimum of 10 hours of class time.

Each MathsCraft Curriculum includes:

- A carefully curated set of problems[1] to present to a whole class of students, working in groups of 2 or 3
- Support resources, including:
    - Videos accompanying some of the problems, demonstrating how the adventure might unfold
    - A written document, for the rest of the problems, demonstrating how the adventure might unfold
    - A web-based discussion list for the use of all teachers running the MathsCraft Curriculum
- A "Description of Mathematical Behaviours" to help teachers to mentor-by-modelling and for use with assessment
- Certificates of achievement[2] for each student, indicating their group's grade.

EXPECTED STUDENT OUTCOMES

The students will:

- Explore mathematics
- Have independent ideas in order to explore, solve, create and understand
- Share ideas and collaborate mathematically
- Develop an appreciation for logical arguments
- Learn to build logical arguments
- Learn to reason in order to become sure of an idea, rather than relying on an authority
- Learn to write down what they are thinking
- Appreciate "dead ends" as a positive step in their adventure
- Develop their critical and creative thinking
- Develop their resilience and persistence

[1] There will be sufficient problems to enable students to participate in each MathsCraft Curriculum for multiple years.
[2] Certification is provided by ACEMS — a federally-funded Centre of Excellence for mathematical research, comprising researchers from University of Adelaide, University of Melbourne, Monash University, University of New South Wales, University of Queensland, Queensland University of Technology and University of Technology Sydney.
ACEMS has funded the development of the MathsCraft program since 2015.

> *'[It] was exciting to see that once they realised it was about offering suggestions/ observations and questions how the level of interaction increased. Their usual teacher commented on how she liked the process of testing, refining and moving forward'*

'The [students] have developed resilience and risk taking and the idea that there may not be a final, and only one, answer. What a great project'

> *'My students are doing a compulsory investigation now and I am already seeing the benefits of the MathsCraft programme — they aren't complaining about being lost so much, and they are careful to see that a pattern **always** holds, not just for a few cases — MathsCraft fruits!'*
> [Sent one month after the completion of the curriculum]

'[We] could not bring ourselves to clean the board. The girls left with a real buzz and we could hear them still saying: "but what if …" '

> *'I loved it, despite the fact that I had a really difficult class ... [with a] generally negative attitude to mathematics ... I really liked the growth that I saw in the students over the sessions — they learned to collect data and make mathematical observations, as well as embrace the "dead ends" as being valuable things to learn from. They rose to the challenges I gave them many times, and were generally engaged with the process'*

Teachers reported increased enjoyment in their teaching by spending some class time *doing mathematics*.

STRUCTURE OF THE MATHSCRAFT CURRICULUM

SCHEDULE

The curricula are designed to be implemented during the school year on a regular basis — e.g., one MathsCraft session per week or per fortnight for one or two terms.

WORKING ARRANGEMENTS

Students work in groups. Three is the preferable group size.
Students work in the same group for the entirety of the curriculum (exceptions can be made in special circumstances – please contact us in this case).

TIME COMMITMENT

MathsCraft Curriculum B requires a minimum of 10 hours of class time, during the school year — about one 1-hour session every second week. MathsCraft Curriculum A requires a minimum of six hours of class time over the same period.

This class time is to be used to introduce at least six MathsCraft problems, that will each spark an adventure. Of course, a teacher can choose to spend more than the specified time providing opportunities for MathsCraft adventures.

PROBLEMS

A carefully curated set of problems is provided.

At least six problems are presented that comprise:

- at least five problems chosen by the teacher from the problem set (provided separately); and
- one final problem, common to all groups that undertake the MathsCraft Curriculum; called the *Asssessment Problem*.

The Assessment Problem is sent to teachers towards the end of program. The Assessment Problem is discussed further under Assessment.

Each problem should be allocated a total time of approximately:

- 60 minutes for Curriculum A,
- 100 minutes for Curriculum B.

Depending on how the adventure sparked by a problem goes, you may wish to slightly extend or reduce the time allocated.

An adventure sparked by one problem can take place over multiple sessions.

Each problem will come with either a video or a written document demonstrating how the adventure might unfold. For further support we offer a variety of professional development activities.

MATHEMATICAL BEHAVIOURS

There are some things a mathematician might do when faced with an interesting problem to ponder and have ideas about. Appendix A (page 16) contains a detailed (though by no means complete) description of such mathematical behaviours.

These behaviours are, in part, the content of the MathsCraft Curriculum. The description of these behaviours helps teachers to mentor-by-modelling and are used for assessment.

A suitable summary of these behaviours is:

- Creating data[3] that become the stimuli for ideas
- Having an idea (or two or three)
- Pursuing the idea to test its worth:
    - Rejecting that idea
    - Modifying that idea
    - Working to become sure, beyond all doubt
- Creating a new problem that builds from the one just studied

One might think about these as four phases of a MathsCraft adventure:

1. calculative phase
2. inductive phase
3. deductive phase and
4. extension phase

While these are numbered, a mathematician's adventure may jump around, quite a lot, between these phases. Often, the adventure is anything but linear.

Some of these behaviours will occur naturally, and some come by being modelled and practised.

SUPPORT

A web-based discussion list is available for the use of all teachers running the MathsCraft Curriculum.

[3] We use data here to mean anything that can be used to stimulate an idea. This could be a set of examples, cases, calculations, etc

MATHSCRAFT SESSIONS

JOB OF THE TEACHER

The teacher's job is to present the problem and assist the flow of the adventure.

Assisting the flow of the adventure, in this curriculum, requires you to do as little as possible, but as much as needed, to create the environment in which students have the chance to *have an idea*.

You can do almost anything except have the idea for them.

Problems can remain unsolved, and questions that arise can remain unanswered, but you may wish to reach some satisfying points before finishing the session.

An example of how a session might be run:

- you present the problem
- groups work for a time
- you pick some groups' ideas for them to share, to act as further stimulus for the others, or to connect to other ideas
- groups work a bit more
- etc.

The video or written document accompanying each problem can provide some scaffolding for the teacher to assist the flow of the adventure.

JOB OF THE STUDENT GROUP

The job of the student group is to work together on the problem, have ideas and pursue them, think of questions to ask, and generally try to get to the heart of what is going on — that is, *do mathematics*.

Students may be called upon to share an idea with the rest of the class.

As the groups work on each problem, they should keep a written account of their adventure.

WRITTEN ACCOUNTS

Their written account[4] should not be a polished solution, but rather a "live recording" of their adventure. This should include:

- what they're thinking and doing
- what they try, what fails, what works
- patterns they see, connections they make
- questions that occur to them
- things that puzzle them
- discoveries made

[4] Writing down their thinking as they go is a skill that is separate to mathematical skills. Students will need to practise this during the classes.

If they try something that doesn't work, they should not erase it or cross it out, but simply indicate that it didn't work — such “dead ends” are often as valuable to the end-goal as the great moments themselves.

The written account does not need to be grammatically correct nor mathematically perfect, but it should be clear enough to be understood by a maths teacher who wasn’t present at the time.

If an adventure takes place over multiple sessions, written accounts should be collected by the teacher at the end of a session and given back at the start of the next session.

Various forms of written accounts are acceptable. Samples of various forms are available for viewing.

ASSESSMENT

The assessment in the MathsCraft curriculum is not based on mathematical results, but on mathematical behaviours.

GRADES

The teacher assesses the work of a group by observing and collecting evidence of how the group behaves mathematically.

Appendix A: Description of mathematical behaviours (page 16) is to be used to decide on a grade for each group. The list should be treated as a guide. Not all behaviours are needed for every problem, and some worthy behaviours may not be listed[5].

The grade is a group-grade.

Grades assigned by teachers are moderated by the MathsCraft moderators, after which certificates are provided to each student.

The grades are:

Budding **MathsCrafters**	Generates data and ideas
Competent **MathsCrafters**	As above, and also pursues (some) ideas and critiques them
Proficient **MathsCrafters**	As above, and also applies careful analysis, making logical arguments to support ideas, making progress towards being sure beyond all doubt

EVIDENCE

The main source of evidence for the assigned grade is the group's written account of their adventure sparked by the Assessment Problem.

However, if a group under-performs with the Assessment Problem, relative to your gauge of their performance in earlier problems, written accounts of other adventures can be used as supplementary evidence[6].

In particular, if during an adventure sparked by a teacher-selected problem (i.e. not the Assessment problem) a group successfully behaves as Proficient MathsCrafters, their written account of that problem can be used as supplementary evidence for the grade of Proficient MathsCrafters.

[5] Suggestions for new behaviours for the list can be submitted via one of the feedback options on page 15.
[6] The written account of the Assessment problem is nonetheless required for the moderation process.

THE ASSESSMENT PROBLEM SESSION(S)

The job of the teacher in the Assessment problem is no different than in the teacher selected problems: to present the problem and assist the flow of the adventure.

Assisting the flow of the adventure requires you to do as little as possible, but as much as needed, to create the environment in which students have the chance to *have an idea*.

The Assessment Problem's adventure can span one or two sessions.

If the Assessment Problem's adventure takes place over two sessions, written accounts should be collected by the teacher at the end of the first session and given back at the start of the second session and then collected at the end of that session. Students cannot bring any written materials into the second session to be copied into their written accounts.

When assessing the groups, the teacher should take into account how much input they provided for a group to reach a point where they had their ideas.

MODERATION REQUIREMENTS

Please provide the following, for every group:

- Names and year levels of the students in the group
- A summary of the group's mathematical behaviours and a recommendation for their final grade
- A scanned copy of their written account of the Assessment Problem
- Scanned copies of their written accounts of earlier problems that you wish to include as supplementary evidence, if any.

After moderation, certificates for each student will be provided to you within 4 weeks.

SUPPORT TO THE AUSTRALIAN CURRICULUM: MATHEMATICS

As stated earlier, MathsCraft allows students to *do mathematics* with *facts, concepts, processes and associations* with which they are already *operational*. As a result, students are able to *explore, solve, create and understand*.

Accordingly, by engaging with MathsCraft, students will further develop their skills in the Australian Curriculum's Mathematics Proficiencies: **understanding**, **problem solving** and **reasoning**[7]. The Australian Curriculum's descriptions of these Mathematics proficiencies are given below.

By engaging with MathsCraft, students will also develop their skills with the Australian Curriculum's General Capabilities[8]. MathsCraft addresses **all** of the capabilities listed in Critical and Creative Thinking, and a large number of those listed in Personal and Social Capability, Numeracy and Literacy. The Australian Curriculum's description of the Critical and Creative Thinking General Capability is given below. A list of these General Capabilities, with those that are addressed by MathsCraft duly marked, is given in Appendix B, page 18.

Most current curricula include something like the Australian Curriculum's Proficiencies and General Capabilities. For help in determining whether or not MathsCraft can assist in the delivery of your local curriculum, please contact us.

MATHEMATICAL PROFICIENCY: UNDERSTANDING

Students build a robust knowledge of adaptable and transferable mathematical concepts. They make connections between related concepts and progressively apply the familiar to develop new ideas. They develop an understanding of the relationship between the 'why' and the 'how' of mathematics. Students build understanding when they connect related ideas, when they represent concepts in different ways, when they identify commonalities and differences between aspects of content, when they describe their thinking mathematically and when they interpret mathematical information.

MATHEMATICAL PROFICIENCY: PROBLEM SOLVING

Students develop the ability to make choices, interpret, formulate, model and investigate problem situations, and communicate solutions effectively. Students formulate and solve problems when they use mathematics to represent unfamiliar or meaningful situations, when they design investigations and plan their approaches, when they apply their existing strategies to seek solutions, and when they verify that their answers are reasonable.

MATHEMATICAL PROFICIENCY: REASONING

Students develop an increasingly sophisticated capacity for logical thought and actions, such as analysing, proving, evaluating, explaining, inferring, justifying and generalising. Students are reasoning mathematically when they explain their thinking, when they deduce and justify strategies used and conclusions reached, when they adapt the known to the unknown, when they transfer learning from one context to another, when they prove that something is true or false, and when they compare and contrast related ideas and explain their choices.

GENERAL CAPABILITY: CRITICAL AND CREATIVE THINKING

In the Australian Curriculum, students develop capability in critical and creative thinking as they learn to generate and evaluate knowledge, clarify concepts and ideas, seek possibilities, consider alternatives and solve problems. Critical and creative thinking involves students thinking broadly and deeply using skills, behaviours and dispositions such as reason, logic, resourcefulness, imagination and innovation in all learning areas at school and in their lives beyond school.

Thinking that is productive, purposeful and intentional is at the centre of effective learning. By applying a sequence of thinking skills, students develop an increasingly sophisticated understanding of the processes they can use whenever they encounter problems, unfamiliar information and new ideas. In addition, the progressive development of knowledge about thinking and the practice of using thinking strategies can increase students' motivation for, and management of, their own learning. They become more confident and autonomous problem-solvers and thinkers.

[7] https://www.australiancurriculum.edu.au/resources/mathematics-proficiencies/

[8] https://www.australiancurriculum.edu.au/f-10-curriculum/general-capabilities/

FEEDBACK

If you have any suggestions for improvements to the List of Mathematical Behaviours (page 17), or any other part of the curriculum, please let us know.

You have three options for providing feedback:

- Post it to the MathsCraft Community via Canvas Catalog.
- Email Anita directly at anita.ponsaing@adelaide.edu.au
- Give anonymous feedback here: https://forms.gle/nrHCa6nJv1uq5ZNS6

Suggestions will be used to improve the MathsCraft Curriculum for future years.

APPENDIX A: DESCRIPTION OF MATHEMATICAL BEHAVIOURS

The following page provides a list of mathematical behaviours.

This list is not a recipe to be followed.

It is collection of observable behaviours that are employed by research mathematicians (and young learners) when the stimulus of the work dictates. Mathematical adventures are not linear (see Mathematical Behaviours, page 9), and it is possible that some behaviours will not be called upon during a given adventure.

These behaviours become more effective with experience and a feel for what is required. Such experience can be gained by working with other people and picking up on what they do. It is rarely gained by following a recipe.

This list of behaviours, if applied in a formulaic or mechanical manner, may not work. They are a set of guidelines, not rules, and in the words of Ian Stewart (in his foreword to *How to solve it*, George Pólya, 2nd ed, 1990):

"It is inherent in the nature of guidelines that they don't work if you take them too literally."

LIST OF MATHEMATICAL BEHAVIOURS

DOING THINGS TO HELP YOU HAVE AN IDEA

- Generating data — calculating
- Organising the data
- "Playing" around with the data, getting to know it
- Discussing it with someone
- Looking for patterns or connections
- Establishing a notation
- Deciding on a definition
- Trying a different notation
- Changing a definition
- Asking a question
- …

HAVING AN IDEA

- Observing something
- Thinking of a new way to write something
- Making an association between two things (often one being present in the data, but also a *fact, concept, process or association* that comes to mind)
- Noticing/sensing a pattern
- Making a prediction
- …

DOING SOMETHING WITH THAT IDEA

- Testing it against new data
- Sharing it with someone
- Extending it
- Discarding it
- Convincing someone of its validity
- Recognising gaps in logic
- Cleaning up the idea (working towards a precise statement)
- Forming an argument (working towards becoming sure, beyond all doubt)
- Writing a proof
- …

CREATING A NEW PROBLEM

- Looking before/within/beyond for other things of interest
- Changing, adding, or removing a rule
- Asking a new question
- Pondering other problems known to you and considering links
- …

APPENDIX B: AUSTRALIAN CURRICULUM: GENERAL CAPABILITIES

Below is the list of the capabilities in the "Australian Curriculum: General Capabilities"[9]. The ✓ indicates that students can gain practice at this capability through participation in the MathsCraft Curriculum.

CRITICAL AND CREATIVE THINKING

Inquiring — identifying, exploring and organising information and ideas	
Pose questions	✓
Identify and clarify information and ideas	✓
Organise and process information	✓
Generating ideas, possibilities and actions	
Imagine possibilities and connect ideas	✓
Consider alternatives	✓
Seek solutions and put ideas into action	✓
Reflecting on thinking and processes	
Think about thinking (metacognition)	✓
Reflect on processes	✓
Transfer knowledge into new contexts	✓
Analysing, synthesising and evaluating reasoning and procedures	
Apply logic and reasoning	✓
Draw conclusions and design a course of action	✓
Evaluate procedures and outcomes	✓

NUMERACY

Estimating and calculating with whole numbers	
Understand and use numbers in context	✓
Estimate and calculate	✓
Use money	-
Recognising and using patterns and relationships	✓
Using fractions, decimals, percentages, ratios and rates	
Interpret proportional reasoning	✓
Apply proportional reasoning	✓
Using spatial reasoning	
Visualise 2D shapes and 3D objects	✓
Interpret maps and diagrams	-
Interpreting statistical information	
Interpret data displays	-
Interpret chance events	-
Using measurement	
Estimate and measure with metric units	-
Operate with clocks, calendars and timetables	-

[9] https://www.australiancurriculum.edu.au/f-10-curriculum/general-capabilities/

LITERACY

Comprehending texts through listening, reading and viewing	
Navigate, read and view learning area texts	-
Listen and respond to learning area texts	-
Interpret and analyse learning area texts.	-
Composing texts through speaking, writing and creating	
Compose spoken, written, visual and multimodal learning area texts	✓
Use language to interact with others	✓
Deliver presentations	✓
Text knowledge	
Use knowledge of text structures	-
Use knowledge of text cohesion	-
Grammar knowledge	
Use knowledge of sentence structures	-
Use knowledge of words and word groups	-
Express opinion and point of view	✓
Word knowledge	
Understand learning area vocabulary	✓
Use spelling knowledge	-
Visual Knowledge	
Understand how visual elements create meaning	✓

PERSONAL AND SOCIAL CAPABILITY

Self-awareness	
Recognise emotions	-
Recognise personal qualities and achievements	-
Understand themselves as learners	✓
Develop reflective practice	✓
Self-management	
Express emotions appropriately	-
Develop self-discipline and set goals	✓
Work independently and show initiative	✓
Become confident, resilient and adaptable	✓
Social awareness	
Appreciate diverse perspectives	✓
Contribute to civil society	-
Understand relationships	-
Social management	
Communicate effectively	✓
Work collaboratively	✓
Make decisions	✓
Negotiate and resolve conflict	-
Develop leadership skills	-